# Consistent Mesh Parameterizations

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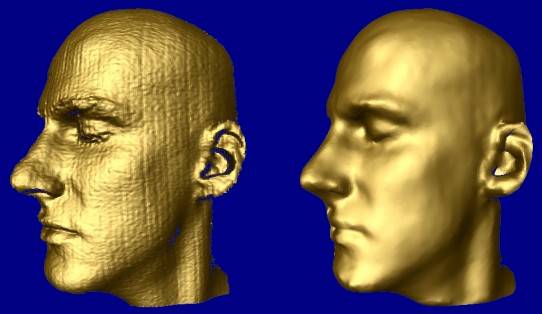
# Motivation

## Digital Geometry Processing (DGP)

- Do for surfaces what DSP does for sound, images, and video



denoising

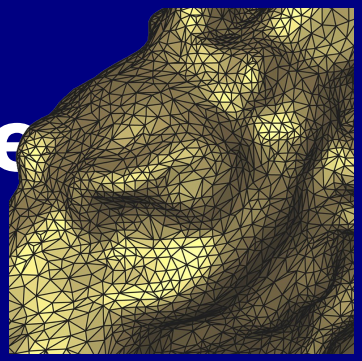
enhancement

- Requires smooth parameterizations

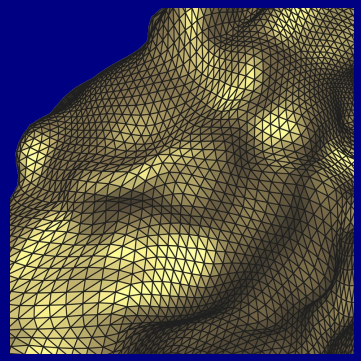
# Parameterizations

## Smooth sampling patte

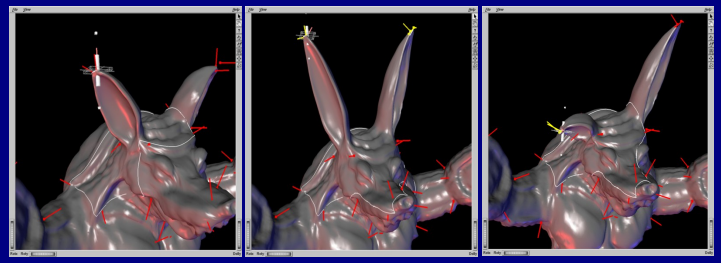
- Individual surface setting
  - *coarse mesh (base domain)*
  - *semi-regular refinement*
- Efficient algorithms



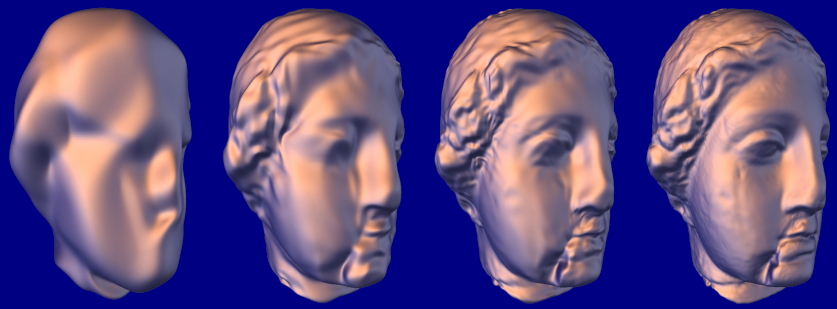
irregular



semi-regular



hierarchical editing

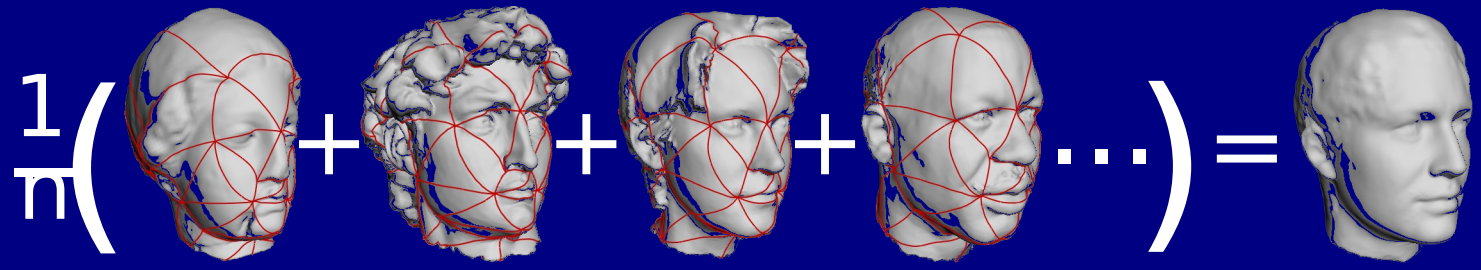


progressive transmission

# Parameterizations

**What about multiple objects?**

- Computing the mean




- ... and many other algorithms
  - *blending, principal components, etc.*

**Need consistent parameterizations!**

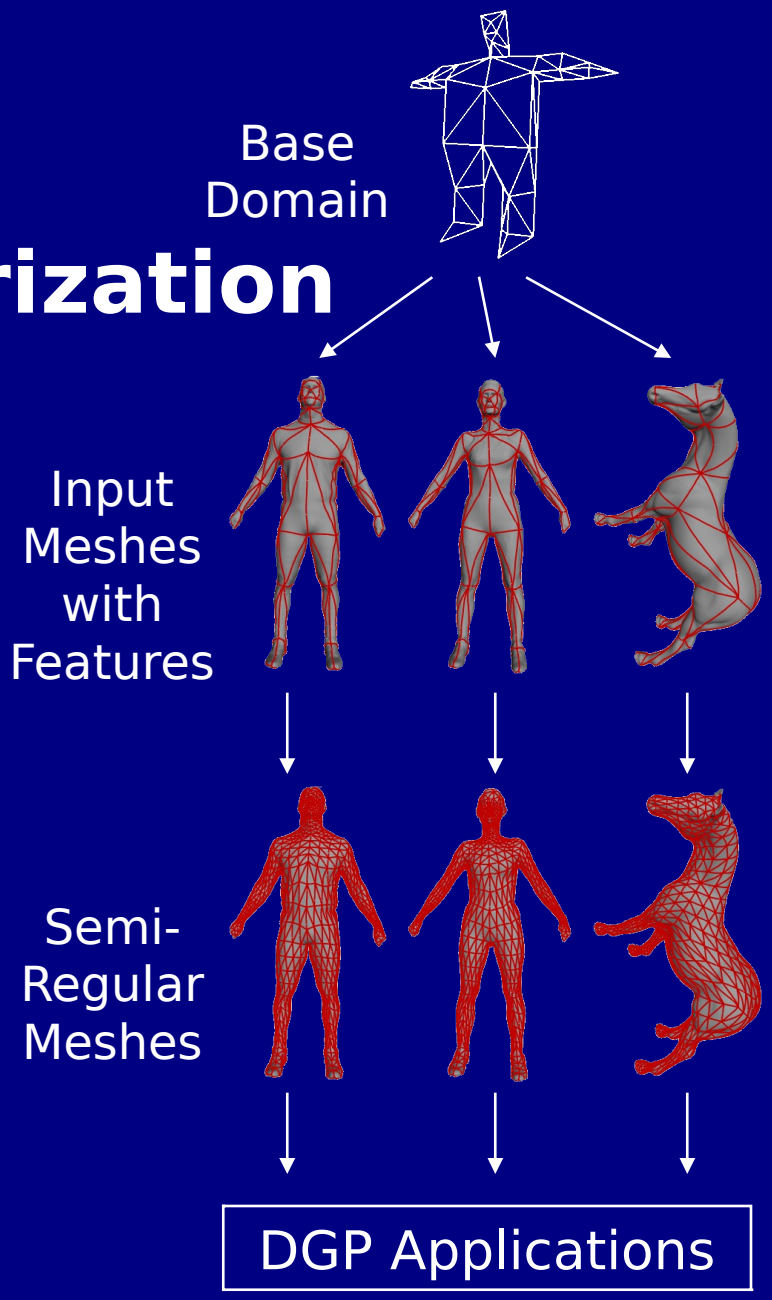
# Goal

## Consistent parameterization

- same base domain
- correspondences
  - *vertices, edges*
- smooth & fair

## Common sampling

- samples 1-1



# Previous Work

**Mesh Simplification, Progressive Meshes, …**

- [Hoppe 94-98]

**MAPS, Morphing**

- [Lee 98, 99]

**Disp. Subdivision Surfaces / Normal Meshes**

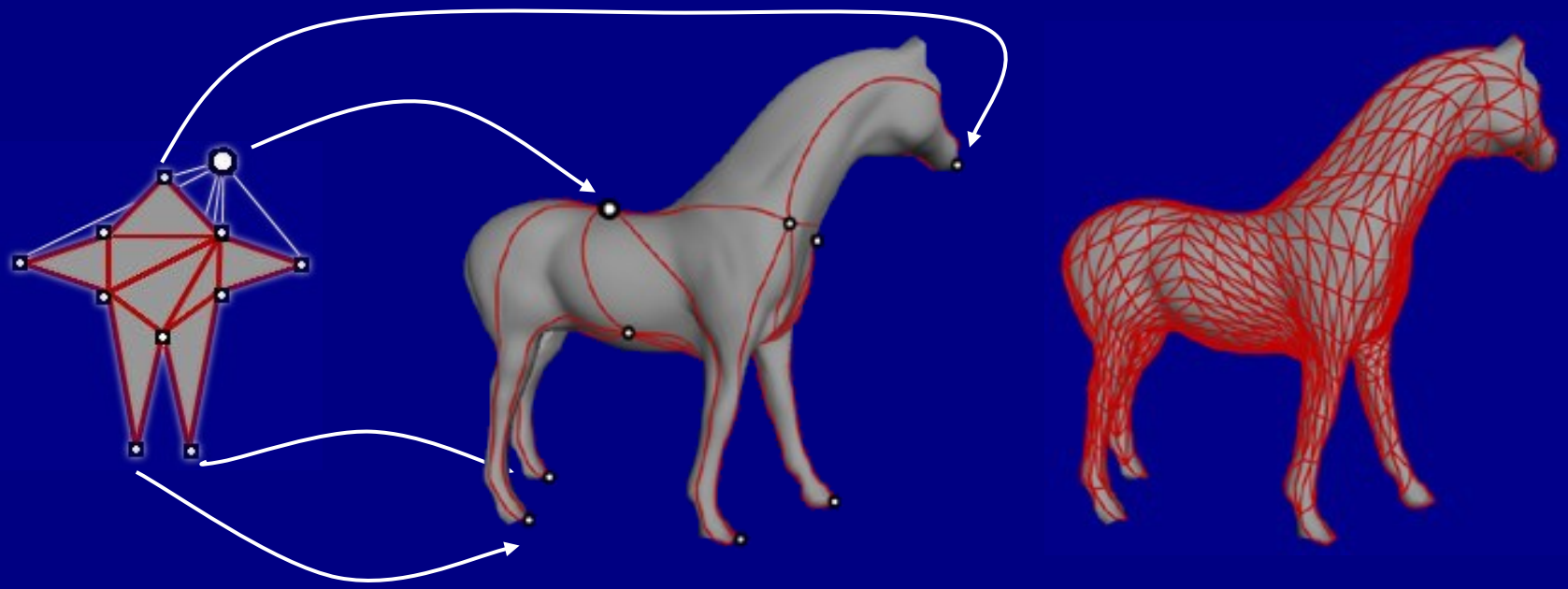
- [Lee 2000] / [Guskov 2000]

# Approach

**Identify feature points (user)**

**Trace curves for base domain edges**

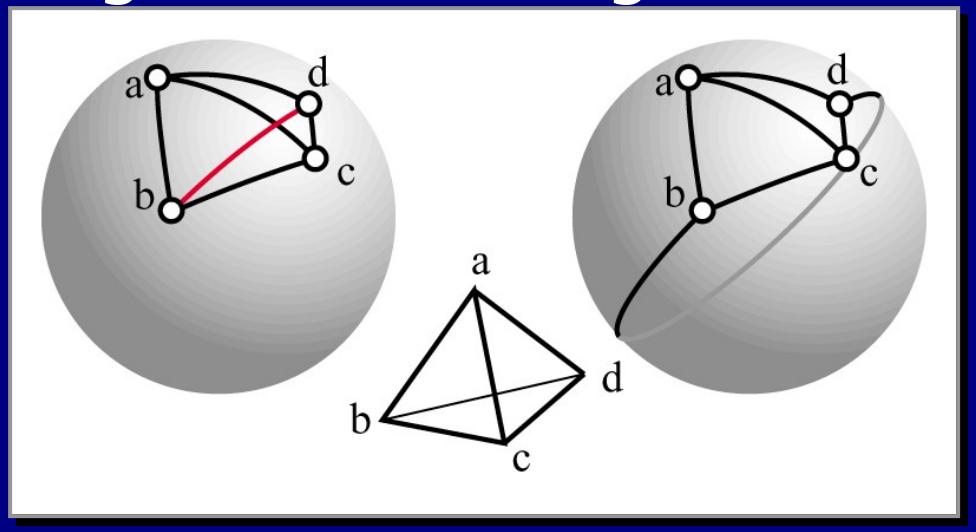
**Parameterize interior of patches**



# Tracing Curves

**Net topologically equivalent to base domain**

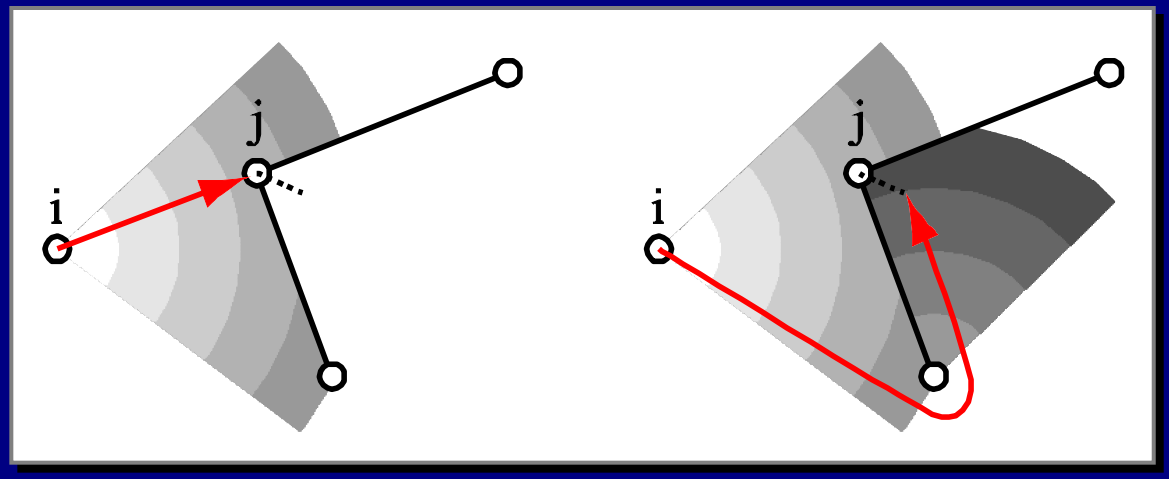
- Curves intersect only at vertices
- Same neighbor ordering around vertices



# Tracing Curves

**Restricted “brush fire” (BFS traversal):**

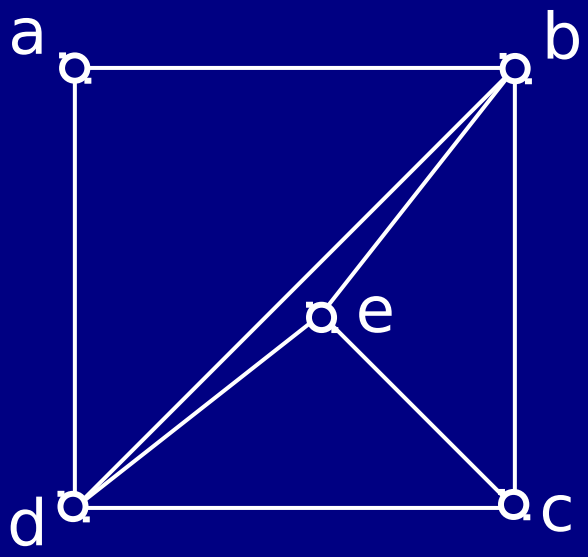
- Do not cross other curves
- Start & end in correct sector



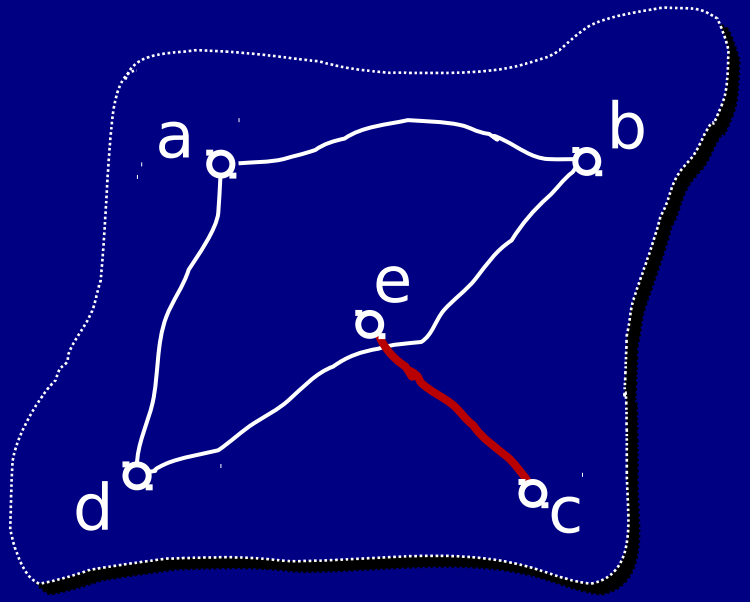
# Problem: Encircling

**To avoid, first trace spanning tree**

**Proof of correctness in the paper**




Base domain



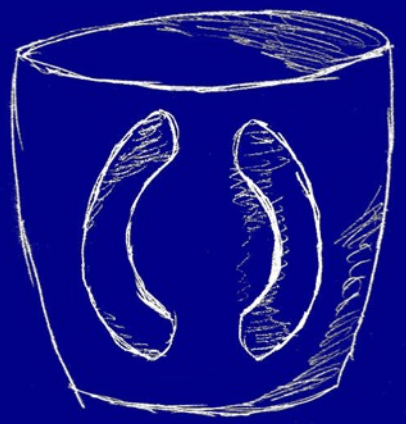

Mesh

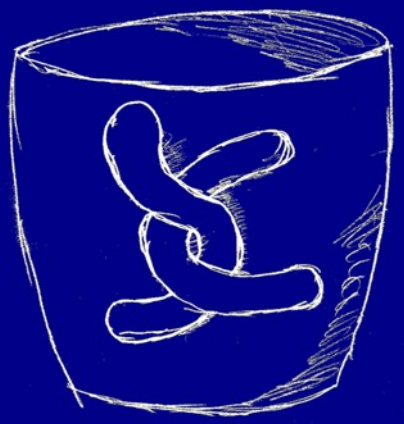
# Topological Equivalence

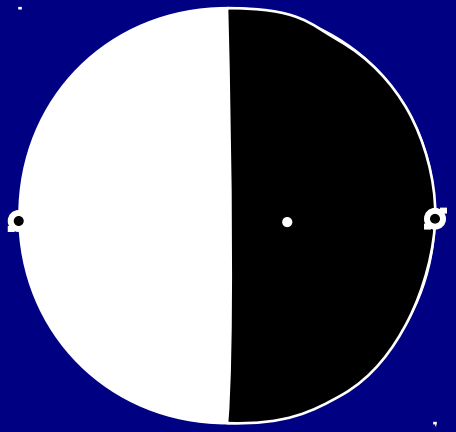
**Guarantee topological equivalence of traced net and base domain**

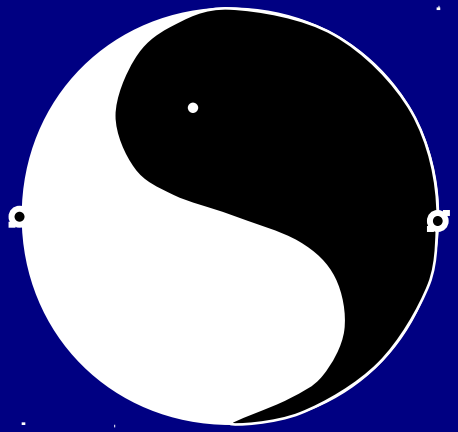
- Trace curves w/ restricted brush fire
- Complete spanning tree before adding cycles

# Is “Topological” Enough?









# Swirl Operator

**Simple relaxation doesn’t undo swirls**

**Infinity of possible configurations**

- We want the least unnecessary swirls
- Optimization very hard; use heuristics

# Heuristics

1. Feature points repel curves
2. Introduce curves in order of length
3. Delay edges of flipped triangles

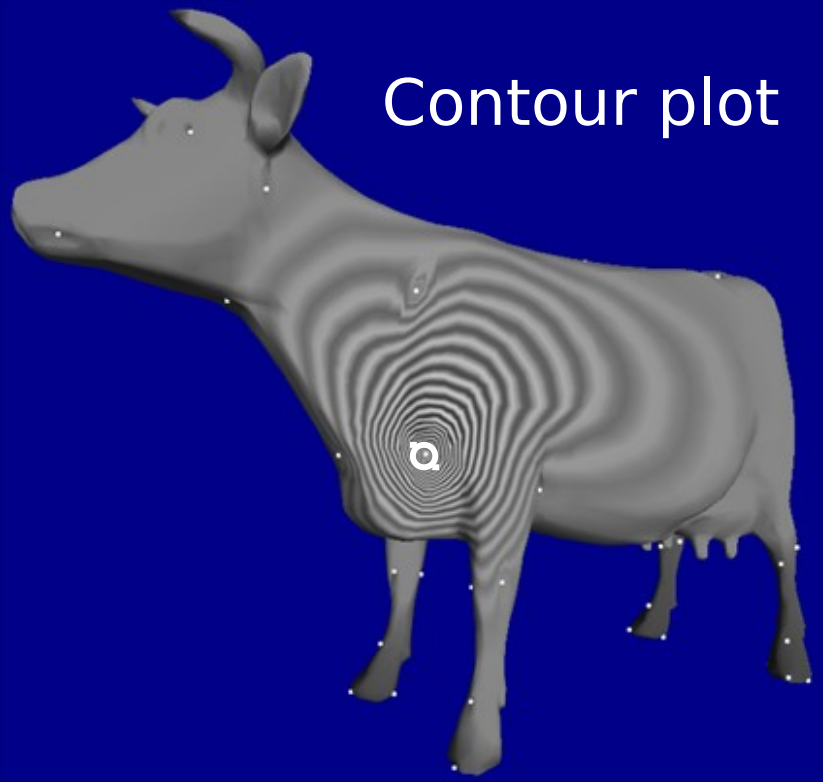
# 1. Features Repel Curves

**Use embedding in**

$\mathfrak{R}^n$

**Compute**

- $I_i(k)$ = “influence” of feature i on vertex k



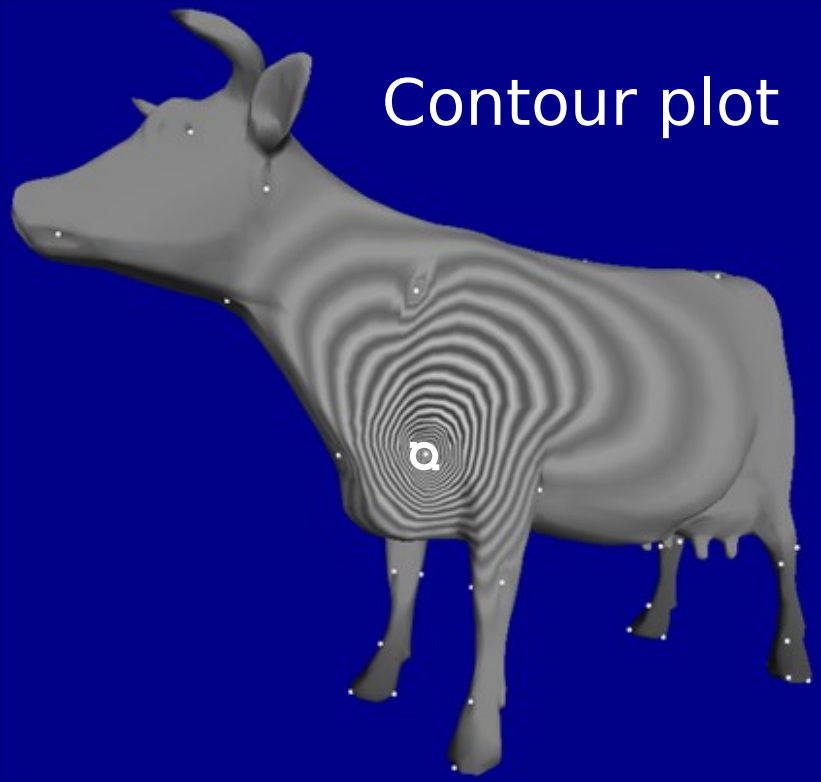
# 1. Features Repel Curves

**Initialize:**

- $I_i(i) = 1$
- $I_i(\text{feature } j) = 0$

**Relax for mesh surface**

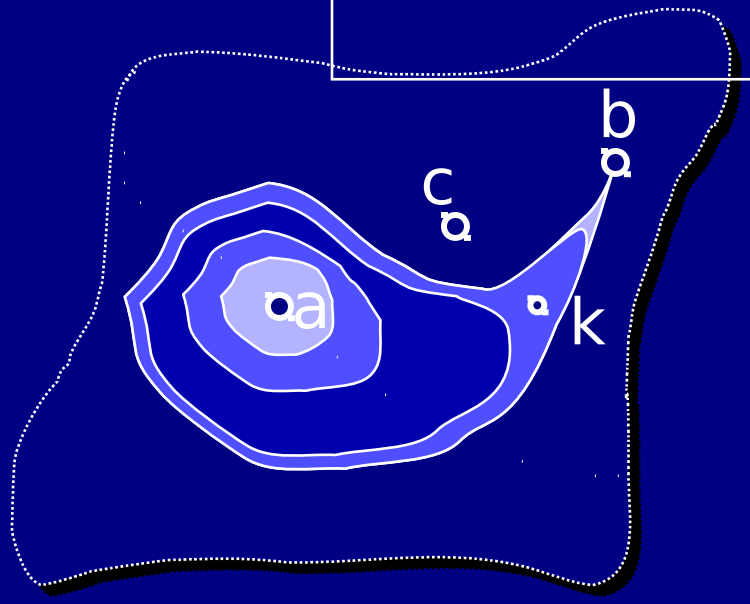
- Linear system
- Floater's weights

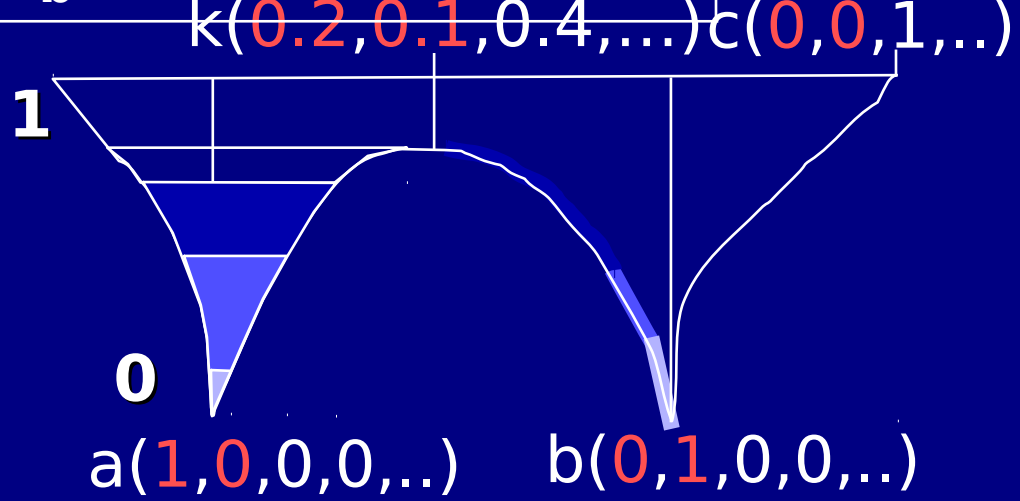


# 1. Features Repel Curves

**Trace curve (a,b): brush fire with variable propagation speed**

**Priority** $P(k) = 1 - I_a(k) - I_b(k)$



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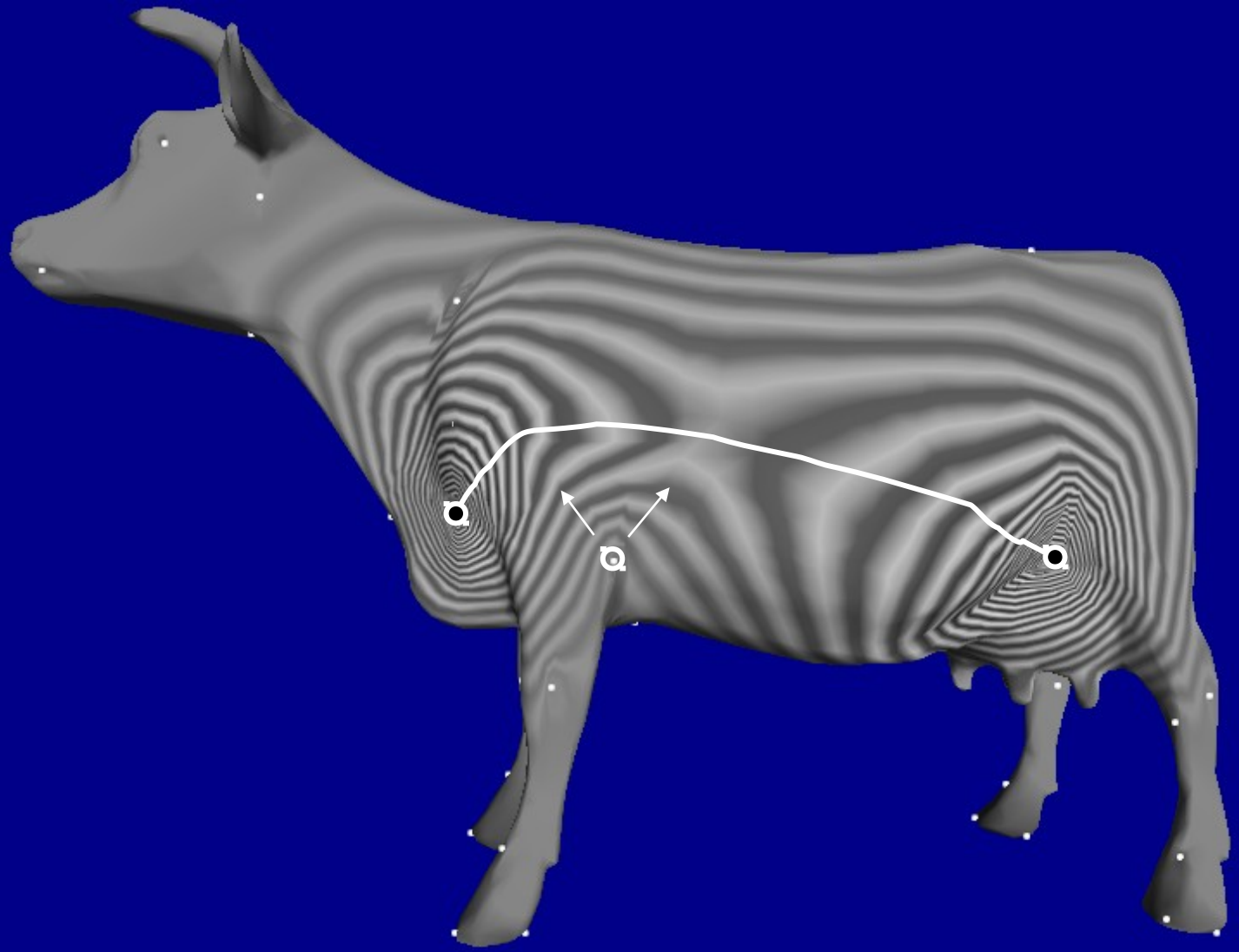


# 2. Prioritize Curves by Length

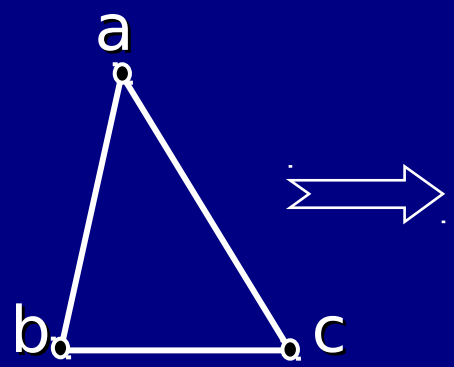
**First stage: complete spanning tree**

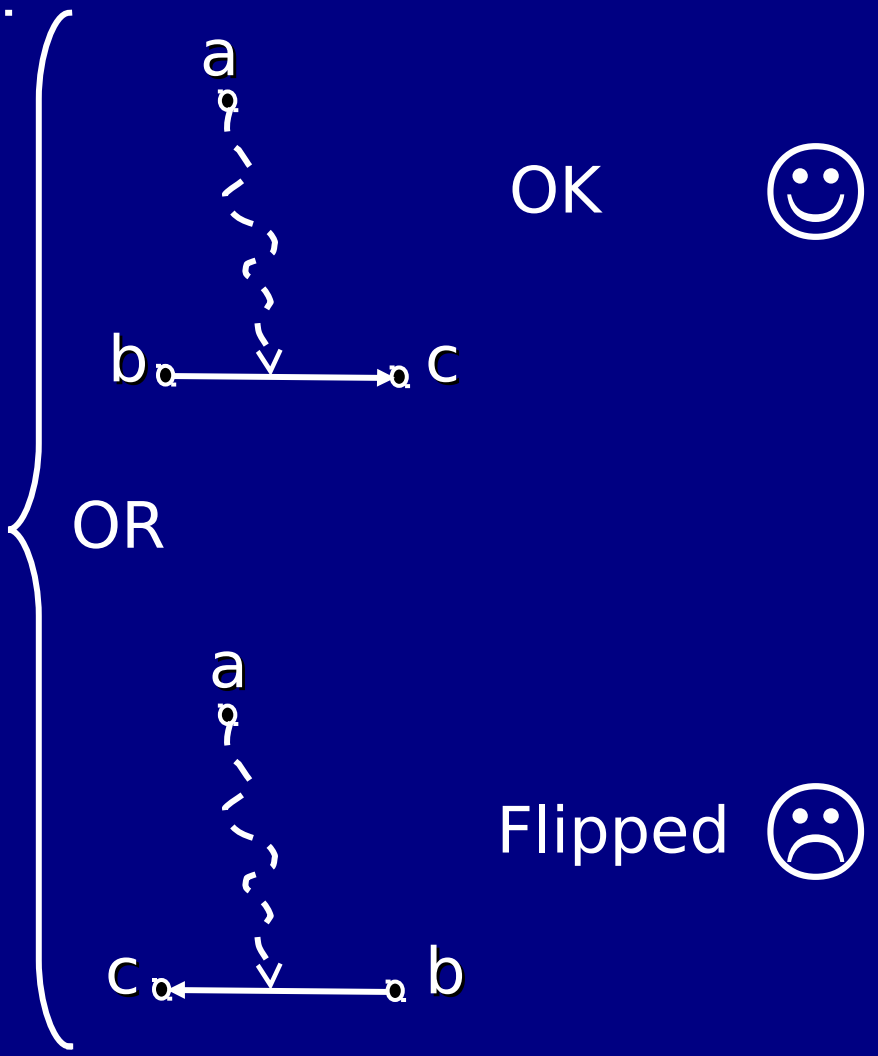
**Second stage: complete whole net**

**For each stage, keep priority queues**

- Queues contain candidate curves
- May need to update to enforce topology

# 3. Delay Edges of Flipped Triangles




**“swirl detector”**

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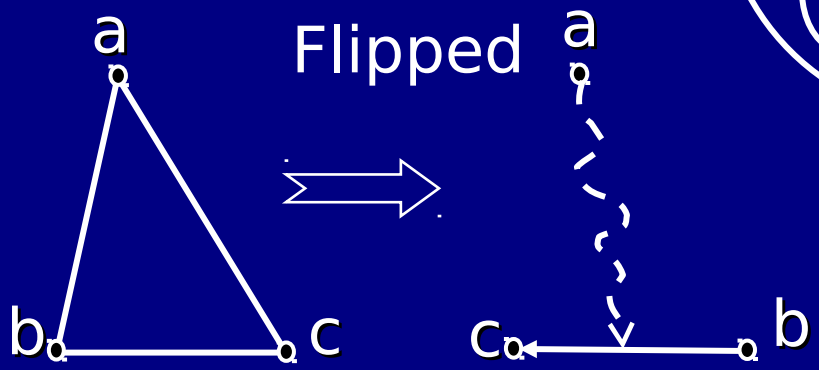
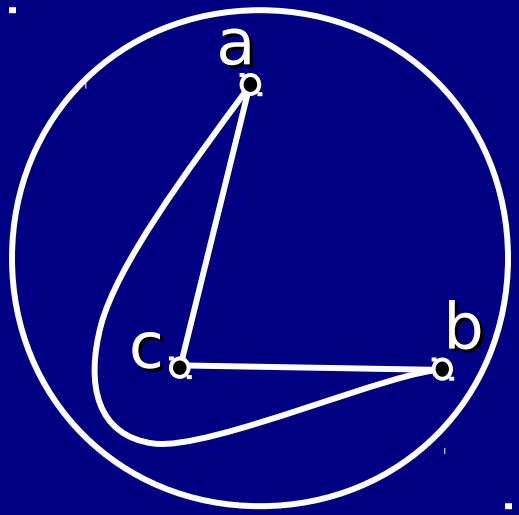
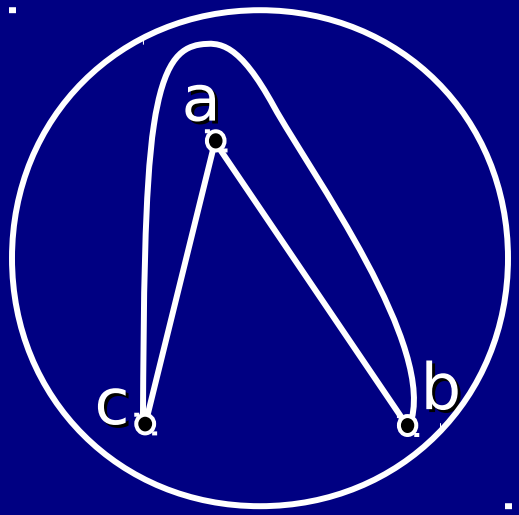
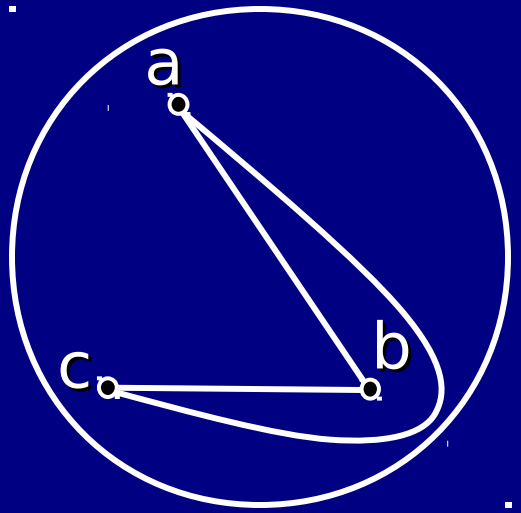


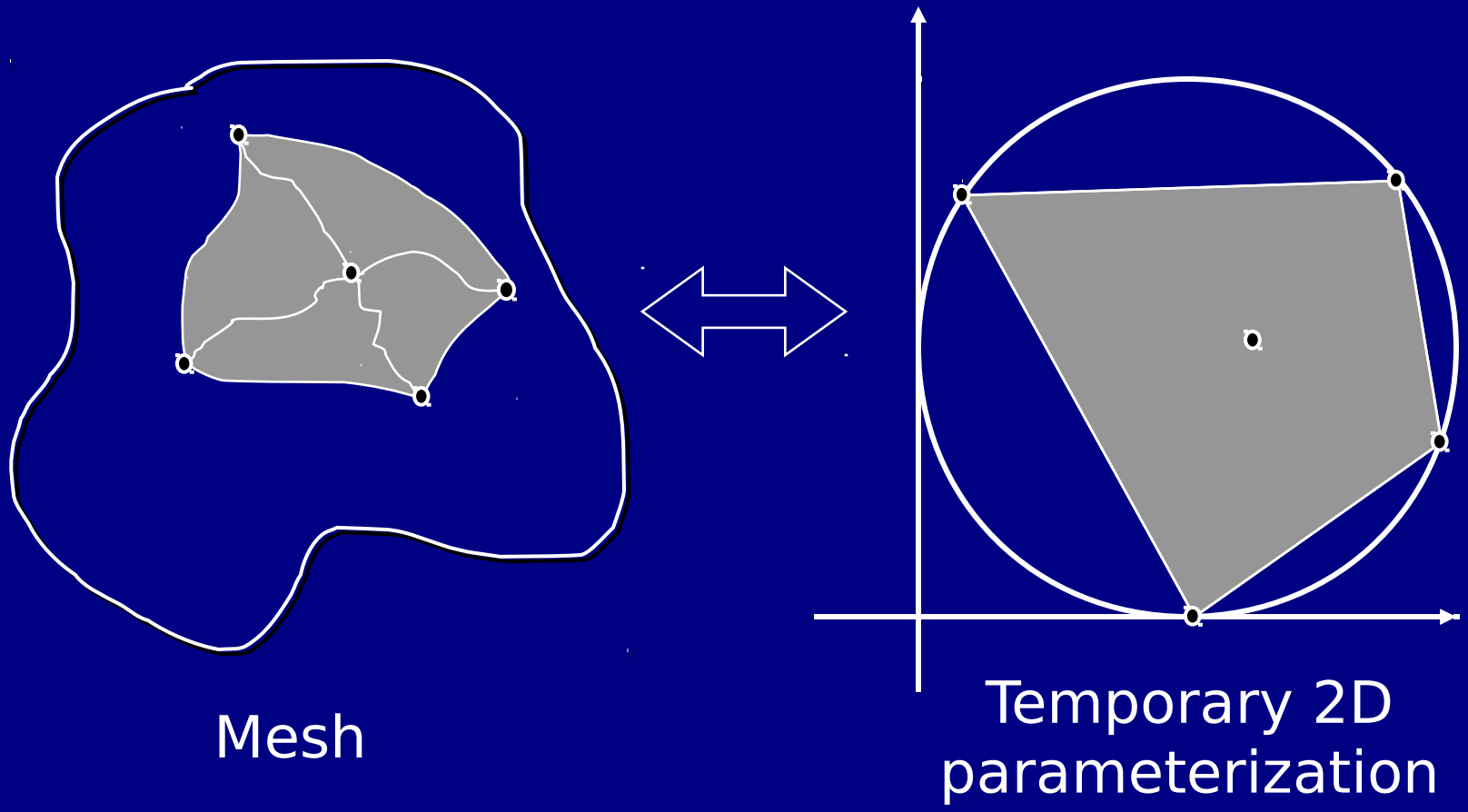

?




**“swirl detector”**

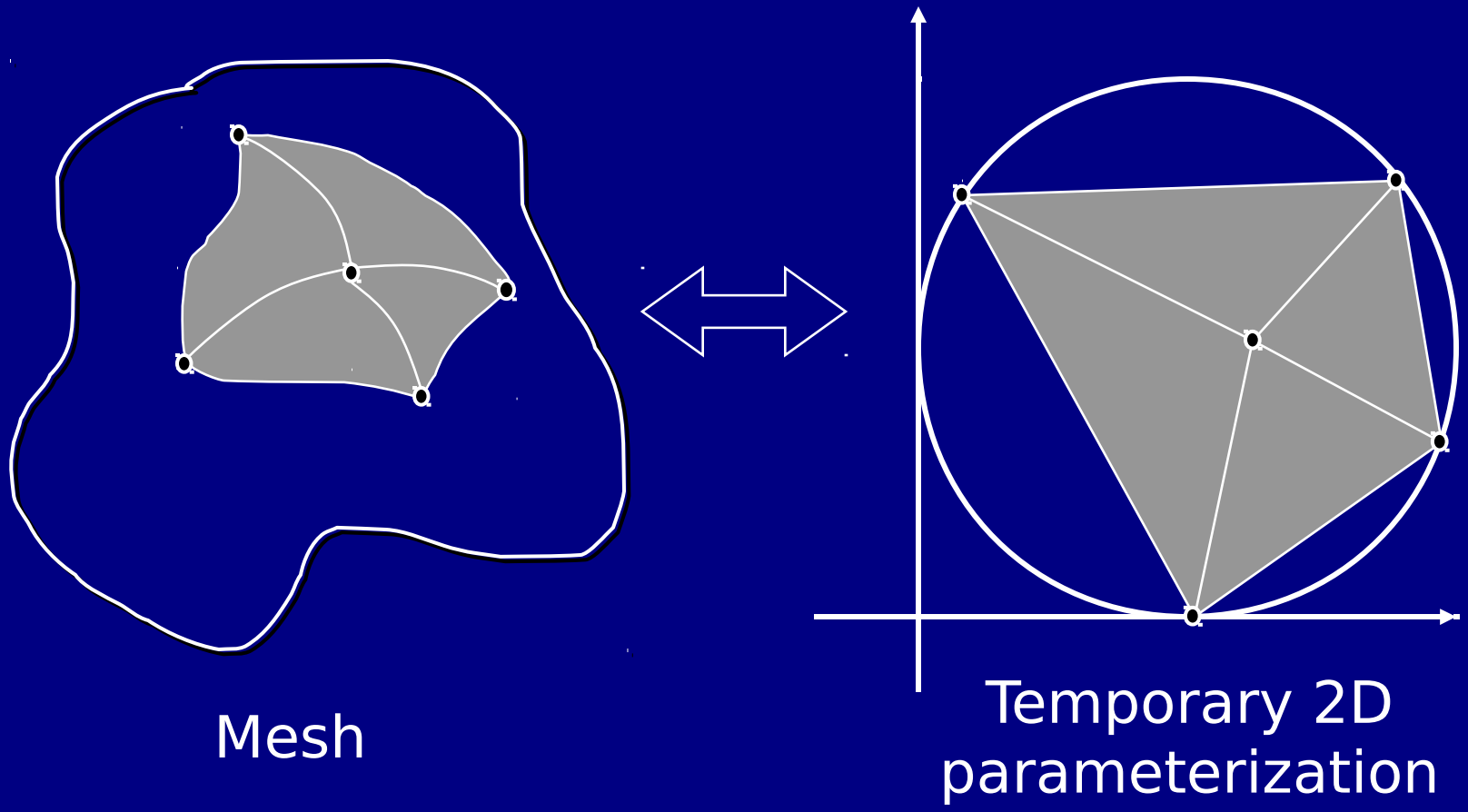
# Edge Straightening



Mesh

Temporary 2D parameterization

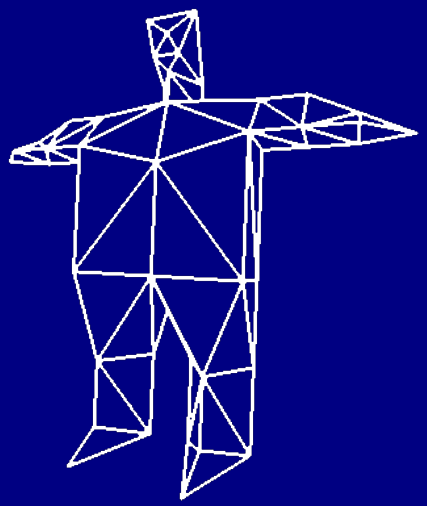
# Edge Straightening

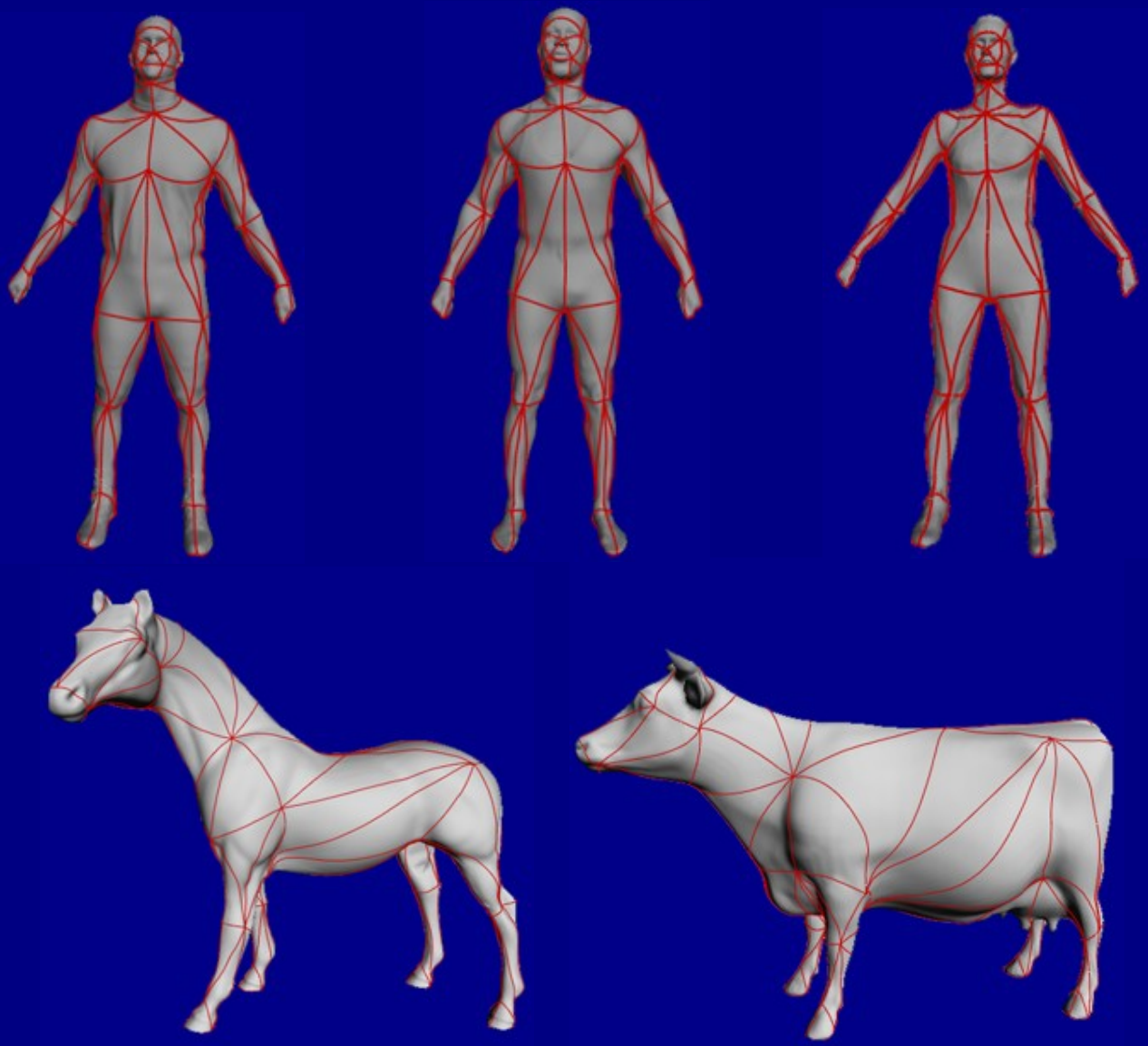


Mesh

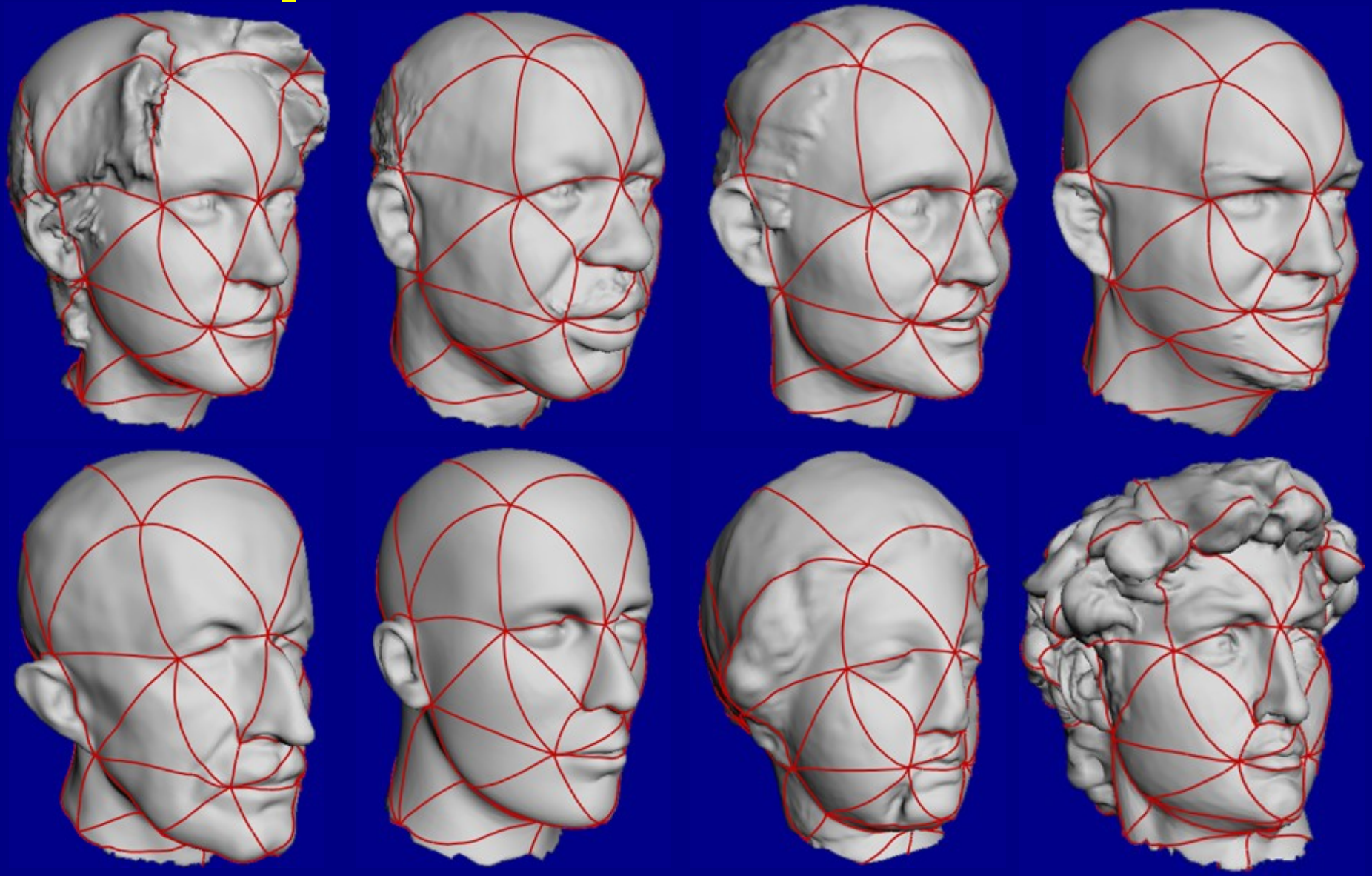
Temporary 2D parameterization

# Examples

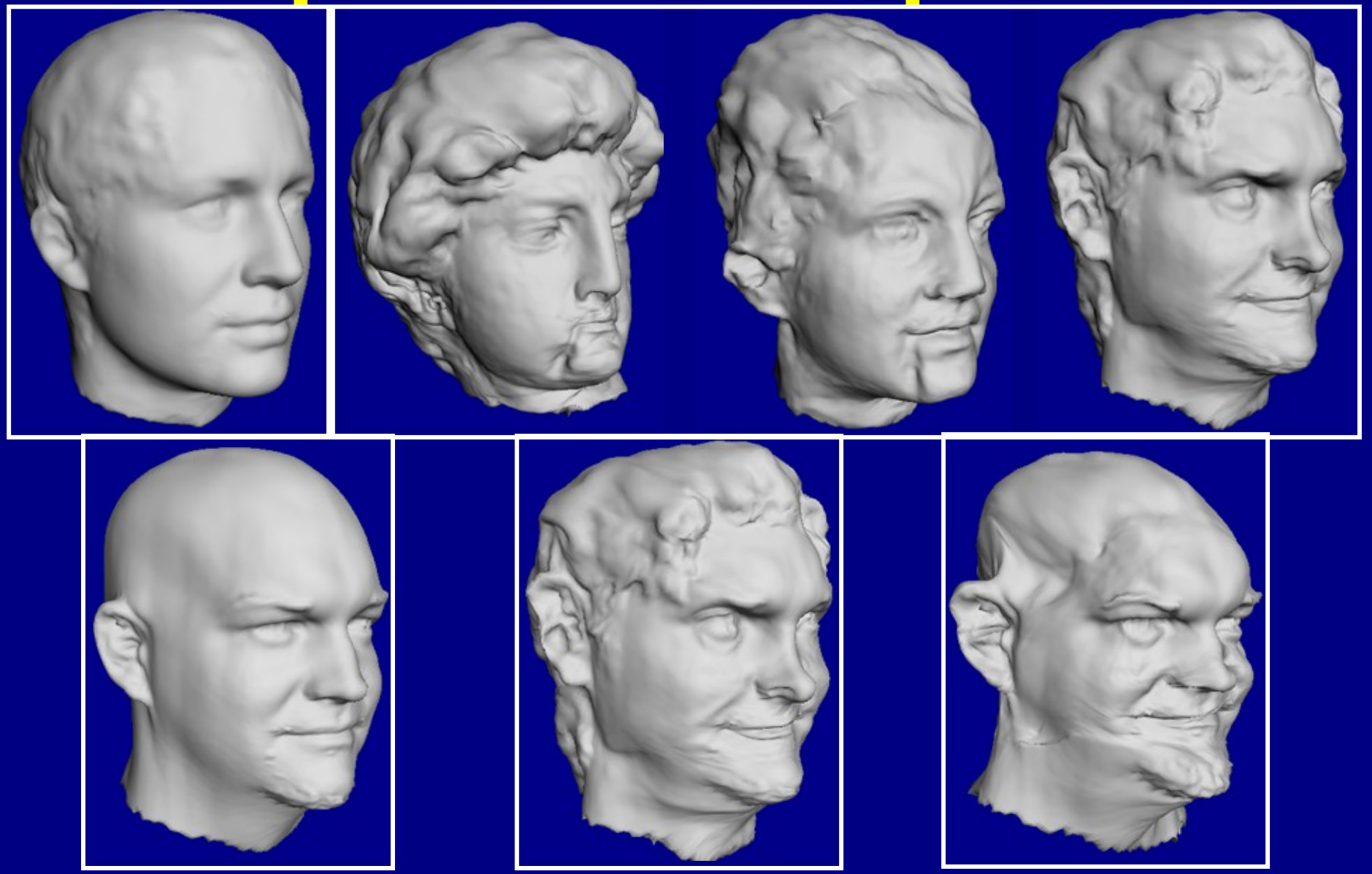




# Examples



# Principal Mesh Components



# Texture Transfer

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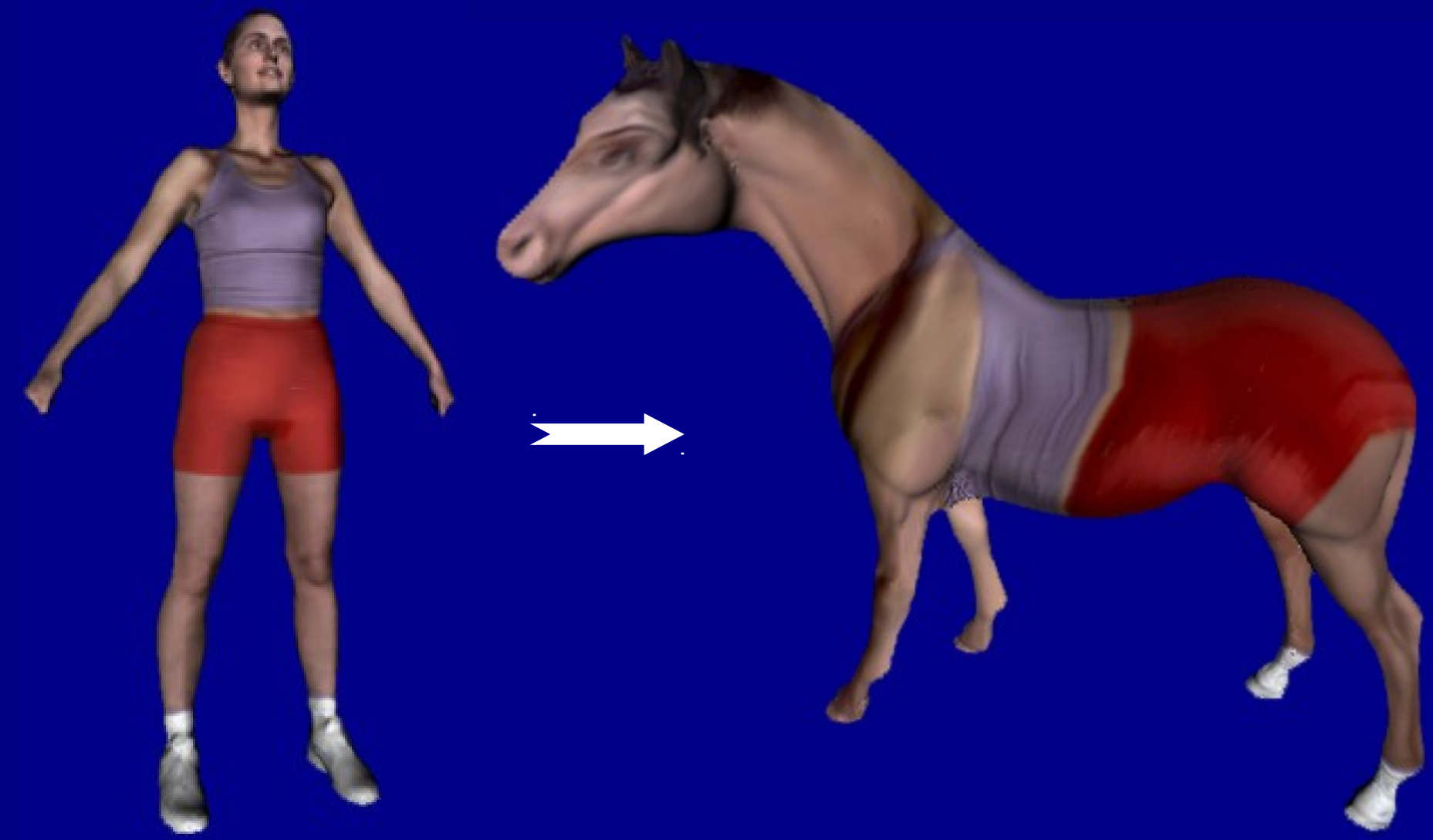
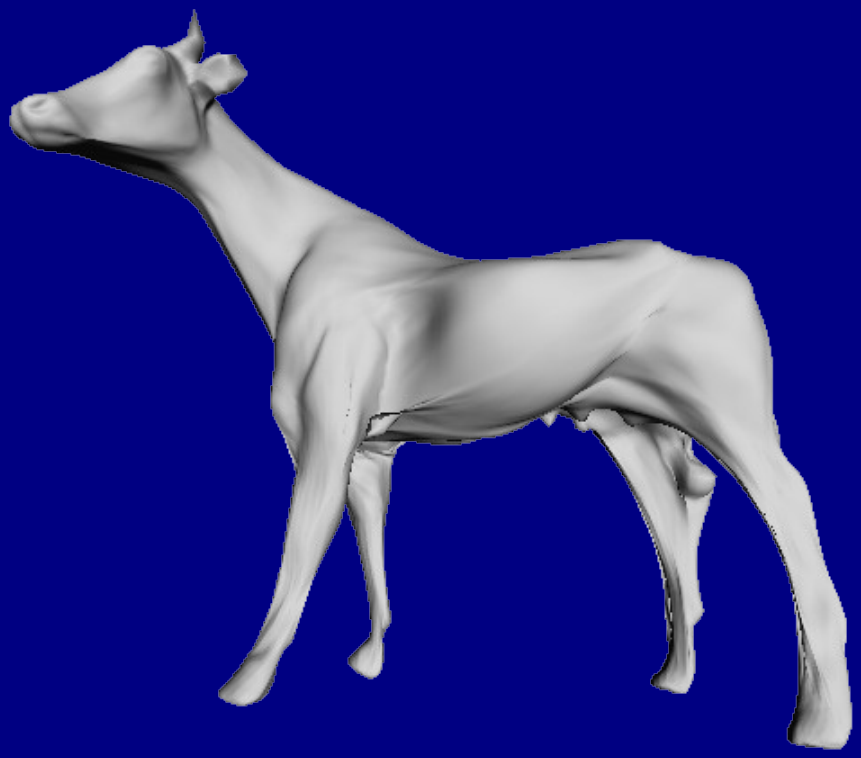
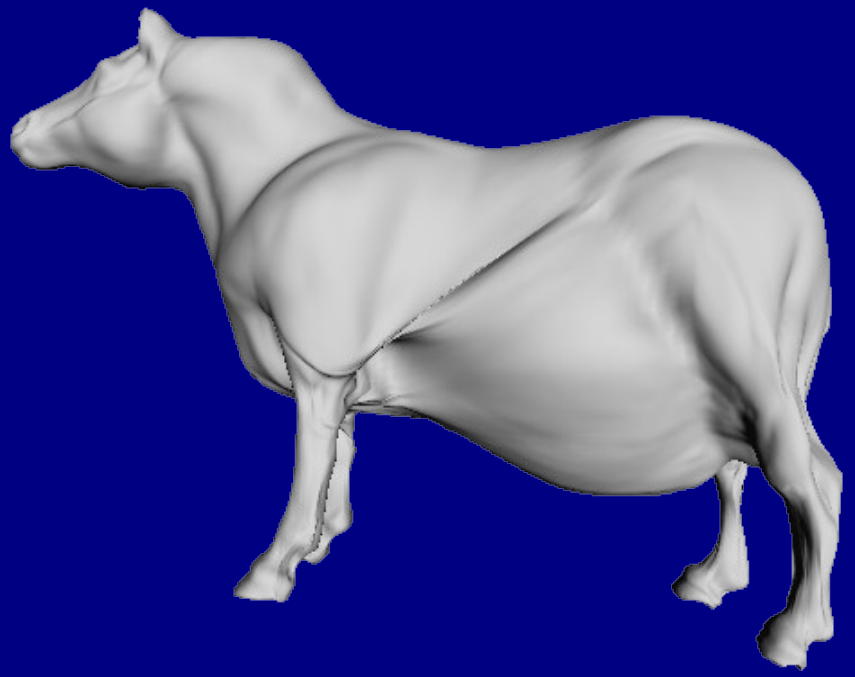


# Detail Transfer



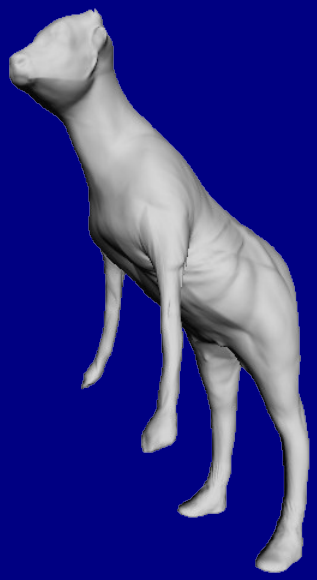


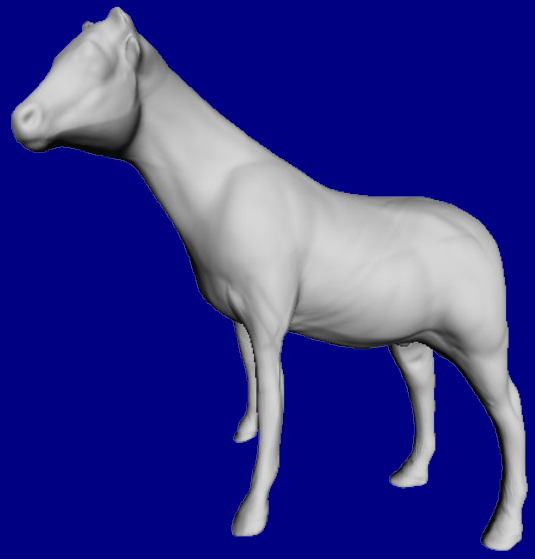
# N-way Shape Blending

Horse .33

Man .33

Cow .33





Horse .5

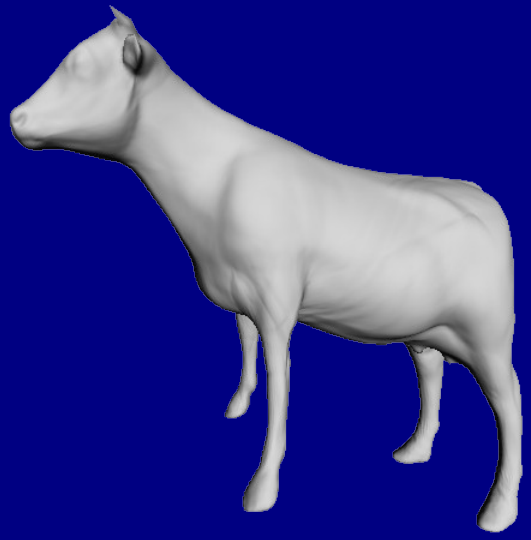
Man .25

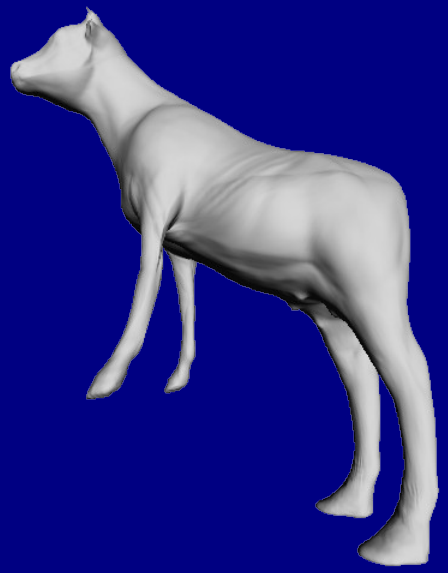
Cow .25

Horse .25

Man .25

Cow .5





Horse .25

Man .5

Cow .25

# Future Work

- Higher genus, boundaries, missing feature points, additional feature points.
- Transfer of animation controls
- Use of principal component analysis for indexing and recognition in large database
- Compression of multiple shapes

# Acknowledgements

**Support: Bell Labs**

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**Code: Igor Guskov**

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**& the Princeton crowd**